GLENTYDE
CAPITAL ADVISORS
P O Box 79091 Charlotte NC 28271-7048 704-614-9555
www.Glentyde.com

January 24, 2018

Summary: Showing the quantitative relationship between marginal revenue and demand elasticity with respect to price.

Typical use case: Finding the unit price(s) at which demand elasticity exactly equals -1, as it's generally understood that marginal revenue (R') equals zero at such point(s), and since *maximum* revenue implies $R' = 0$. (As noted below, though, it's not true that $R' = 0 \Rightarrow$ max revenue.) Depending on availability of industry and sector economic data and studies, it might be easier to obtain elasticity estimates for a given good or service, than to derive an accurate revenue function.

Caveats and notes:

- This memo doesn't adopt the convention of taking elasticity measures as their absolute values. Elasticity values herein will be negative.
- Identifying those points at which marginal revenue is zero isn't enough. Closer examination of the identified "critical points"—say, via second derivatives, or other means if an accurate revenue function isn't available—will be needed to determine if a critical point represents a local maximum or minimum, global max or min, or none of these.
- *Revenue* maximization doesn't necessarily coincide with *profit* maximization.

Notation:

- Q := Quantity (units sold) as function of price; $Q(P)$.
- P := Unit price.
- R := Total revenue, as product of price and quantity; $R(P) = PQ = P \times Q(P)$.
- E := Marginal elasticity of quantity with respect to price, at a given point P_0. Generically, $E =$ $\%$ change in unit demand $\div$ $\%$ change in price.
- The first derivatives R' and Q' are understood to be with respect to P.

Theorem: $R'(P) = \frac{dR}{dP} = 0 \iff E = -1$. That is, marginal revenue (the first derivative of the Revenue function) equals zero iff marginal elasticity equals -1.

Proof:

$$R' = \frac{d}{dP}(PQ) = \frac{dP}{dP}Q + P\frac{dQ}{dP} \quad \boxed{= Q + PQ'}.$$

$$E = \lim_{\Delta P \to 0} \frac{\%\Delta Q}{\%\Delta P} = \lim_{\Delta P \to 0} \frac{\Delta Q/Q}{\Delta P/P} = \lim_{\Delta P \to 0} \frac{\Delta Q}{Q}\frac{P}{\Delta P} = \lim_{\Delta P \to 0} \frac{\Delta Q}{\Delta P}\frac{P}{Q} \quad \boxed{= \frac{Q'P}{Q}}.$$

$$R' = 0 \iff Q'P = -Q \iff \frac{Q'P}{Q} = -1 \iff E = -1$$

Similarly, it's easy to show that

- Elasticity less than -1 ("elastic" situation) implies negative R'; hence a small price *decrease* should induce a revenue *increase*;
- Elasticity between -1 and 0 ("inelastic" situation) implies positive R'; hence a small price *increase* should induce a revenue *increase*.